ECED 4601 Digital Control Systems

Assignment #2 Reference Solution

http://www.jasongu.org/4601/assignments.html

Assignment #2 contains the following problems:

1) Problem B-3-4: Consider a transfer function system

$$X(s) = \frac{(s+3)}{(s+1)(s+2)}$$

obtain the pulse transfer function by two different methods.

Solution

1. <u>Residue method</u>:

$$X(z) = \left[\text{residue of } \frac{X(s)z}{z - e^{Ts}} \text{ at pole } s = -1\right] + \left[\text{residue of } \frac{X(s)z}{z - e^{Ts}} \text{ at pole } s = -2\right]$$

$$= \lim_{s \to -1}\left[(s+1)\frac{s+3}{(s+1)(s+2)}\frac{z}{z - e^{Ts}}\right] + \lim_{s \to -2}\left[(s+2)\frac{s+3}{(s+1)(s+2)}\frac{z}{z - e^{Ts}}\right]$$

$$= \frac{2z}{z - e^{-T}} - \frac{z}{z - e^{-2T}} = \frac{2}{1 - e^{-T}z^{-1}} - \frac{1}{1 - e^{-2T}z^{-1}}$$

$$= \frac{1 + e^{-T}(1 - 2e^{-T})z^{-1}}{(1 - e^{-T}z^{-1})(1 - e^{-2T}z^{-1})}$$

2. <u>Method based on impulse response function:</u>

$$X(s) = \frac{s+3}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{1}{s+2}$$

The inverse Laplace transform of this equation gives

$$x(t) = 2e^{-t} - e^{-2t}$$

Hence

$$X(z) = \frac{2}{1 - e^{-T}z^{-1}} - \frac{1}{1 - e^{-2T}z^{-1}}$$

$$= \frac{1 + e^{-T}(1 - 2e^{-T})z^{-1}}{(1 - e^{-T}z^{-1})(1 - e^{-2T}z^{-1})}$$

2) B-3-8: Consider the difference equation system

$y(k+2) + y(k) = x(k)$, where $y(k) = 0 \; for \; k < 0$

obtain the response $y(k)$ when the input $x(k)$ is a unit-step sequence. Also, obtain the Matlab solution.

Solution:

$$y(k+2) + y(k) = x(k), \qquad y(k) = 0 \text{ for } k < 0$$

Since $x(k)$ is a unit-step sequence, we have

$$X(z) = \frac{z}{z-1}$$

The initial data are

$$y(0) = x(-2) - y(-2) = 0, \qquad y(1) = x(-1) - y(-1) = 0$$

Hence

$$z^2Y(z) + Y(z) = X(z) = \frac{z}{z-1}$$

or

$$Y(z) = \frac{1}{z^2+1} \frac{z}{z-1} = -\frac{1}{2}\left(\frac{1+z^{-1}}{1+z^{-2}} - \frac{1}{1-z^{-1}}\right)$$

from which we obtain

$$y(k) = -\frac{1}{2}\left(\cos\frac{k\pi}{2} + \sin\frac{k\pi}{2} - 1\right) \qquad k = 0, 1, 2, \ldots$$

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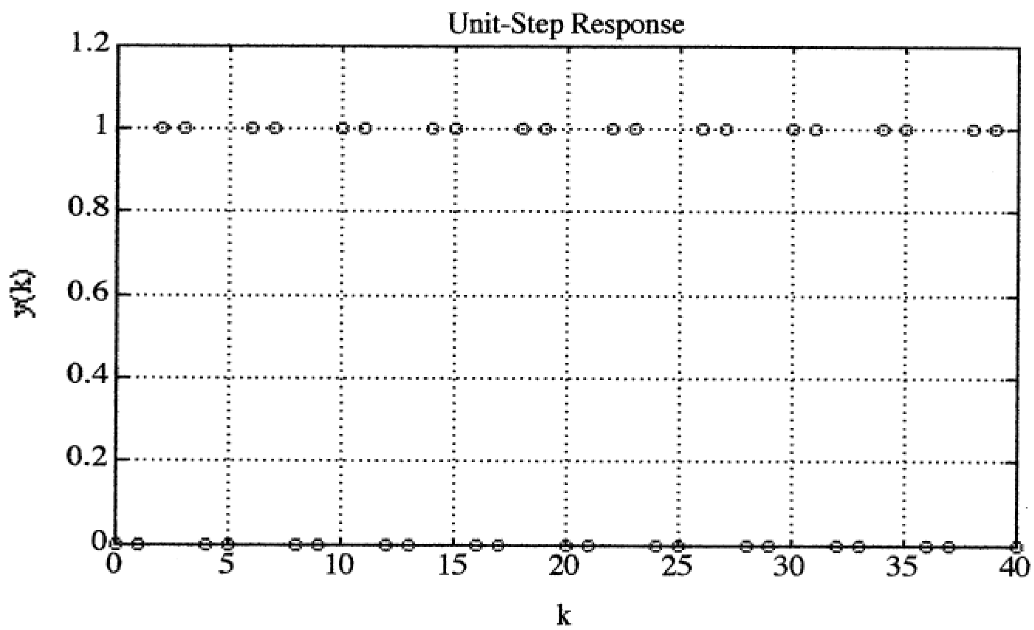
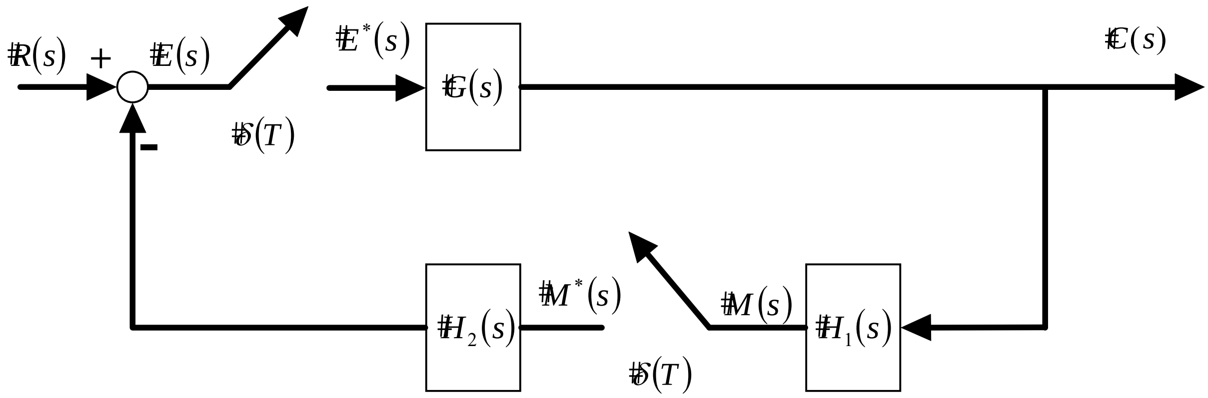
»% MATLAB Program for Problem B-3-8
»
»% ----- Unit-step response -----
»
»num = [0   0   1];
»den = [1   0   1];
»x = ones(1,41);
»v = [0   40   0   1.2];
»axis(v);
»k = 0:40;
»y = filter(num,den,x);
»plot(k,y,'o')
»grid
»title('Unit-Step Response')
»xlabel('k')
»ylabel('y(k)')
```



3) B-3-15 Obtain the closed loop pulse transfer function of the system shown below



Solution:

From the figure we obtain

$$C(s) = G(s)E^*(s)$$

$$E(s) = R(s) - H_2(s)M^*(s)$$

$$M(s) = H_1(s)G(s)E^*(s)$$

By taking the starred Laplace transforms of the preceding equations, we obtain

$$C^*(s) = G^*(s)E^*(s)$$

$$E^*(s) = R^*(s) - H_2^*(s)M^*(s)$$

$$M^*(s) = \left[GH_1(s)\right]^* E^*(s)$$

Hence

$$E^*(s) = R^*(s) - H_2^*(s)\left[GH_1(s)\right]^* E^*(s)$$

or

$$E^*(s) = \frac{R^*(s)}{1 + H_2^*(s)\left[GH_1(s)\right]^*}$$

and

$$C^*(s) = \frac{G^*(s)R^*(s)}{1 + H_2^*(s)\left[GH_1(s)\right]^*}$$

Thus,

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + H_2^*(s)\left[GH_1(s)\right]^*}$$

or

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + H_2(z)GH_1(z)}$$

4) B-3-22

assume that a digital filter is given by the following difference equation:

$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 x(k) + b_2 x(k-1)$

draw the block diagram for the filter using 1) direct programming, 2) standard programming and 3) ladder programming.

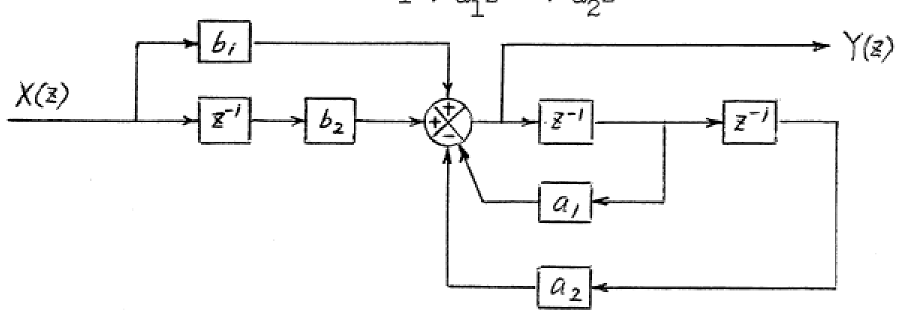
Solution:

1. Direct programming: The z transform of the given difference equation becomes

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_1 X(z) + b_2 z^{-1} X(z)$$

Thus

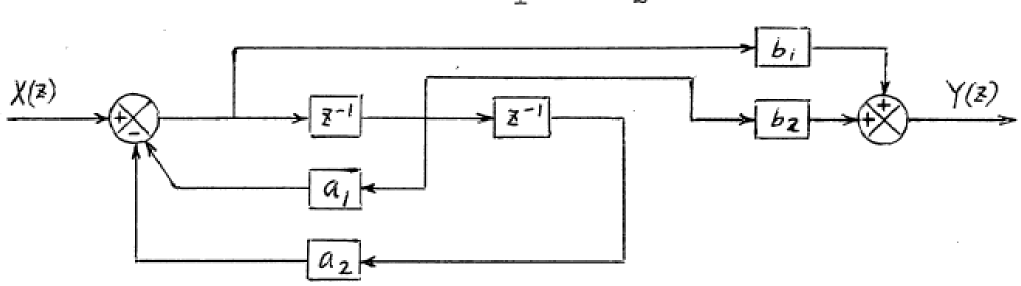
$$\frac{Y(z)}{X(z)} = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$



2. Standard programming:

$$\frac{Y(z)}{H(z)} = b_1 + b_2 z^{-1}$$

$$\frac{H(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$



3. Ladder programming:

$$\frac{Y(z)}{X(z)} = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{b_1 z^2 + b_2 z}{z^2 + a_1 z + a_2}$$

$$= A_0 + \cfrac{1}{B_1 z + \cfrac{1}{A_1 + \cfrac{1}{B_2 z + \cfrac{1}{A_2}}}}$$

where

$$A_0 = b_1$$

$$B_1 = \frac{1}{b_2 - a_1 b_1}$$

$$A_1 = \frac{b_2 - a_1 b_1}{\frac{a_2 b_1}{b_2 - a_1 b_1} + a_1}$$

$$B_2 = -\frac{\frac{a_2 b_1}{b_2 - a_1 b_1} + a_1}{\frac{a_2(b_2 - a_1 b_1)}{\frac{a_2 b_1}{b_2 - a_1 b_1} + a_1} + a_2 b_1}$$

$$\frac{1}{A_2} = \frac{-1}{\frac{b_2 - a_1 b_1}{\frac{a_2 b_1}{b_2 - a_1 b_1} + a_1} + b_1}$$