ECED 4601 Digital Control Systems

Assignment #5 reference solution

http://www.jasongu.org/4601/assignments.html

Assignment #5 contains the following problems:

1) Problem B-6-1: Consider the system defined by
   $x((k+1)) = Gx(k) + Hu(k)$
   $y(k) = Cx(k)$

   Where $G = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix},$

   Determine the conditions on a,b,c, and d for complete state controllability and complete observability.

For complete state controllability, we require

$$\text{rank}\begin{bmatrix} H & \vdots & GH \end{bmatrix} = \text{rank}\begin{bmatrix} 1 & a+b \\ 1 & c+d \end{bmatrix} = 2$$

Thus, the condition is $a + b \neq c + d$.
For complete observability, we require

$$\text{rank}\begin{bmatrix} C^* & \vdots & G^*C^* \end{bmatrix} = \text{rank}\begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} = 2$$

The condition is $b \neq 0$.

2) B-6-8 consider the pulse transfer function system.

$$\frac{Y(z)}{U(z)} = \frac{z^{-1}(1+z^{-1})}{(1+0.5z^{-1})(1-0.5z^{-1})}$$

Obtain the state-space representation of the system in the following forms

a) controllable canonical form
b) observable canonical form
c) diagonal canonical form

1. Controllable canonical form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

2. Observable canonical form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.25 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

3. Diagonal canonical form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} -0.5 & 1.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

3) B-6-12 consider the system defined by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.16 & 0.84 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(k)$$

Determine the state feedback gain matrix such that when the control signal is given by

$$u(k) = -Kx(k)$$

The closed loop system will exhibit the deadbeat response to any initial state $x(0)$

First note that the rank of the controllability matrix is 3 and, therefore, it is possible to determine the necessary state feedback gain matrix K for deadbeat response.

noting that for deadbeat response $\alpha_1 = \alpha_2 = \alpha_3 = 0$, we have

$$K = \left[\alpha_3 - a_3 \;\vdots\; \alpha_2 - a_2 \;\vdots\; \alpha_1 - a_1\right] T^{-1}$$

$$= \left[-a_3 \;\vdots\; -a_2 \;\vdots\; -a_1\right] T^{-1}$$

where

$$a_1 = 0, \qquad a_2 = -0.84, \qquad a_3 = 0.16$$

and

$$T = MW$$

Matrices M and W are given by

$$M = \left[H \;\vdots\; GH \;\vdots\; G^2H\right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0.68 \\ 1 & 0.68 & 0.68 \end{bmatrix}$$

$$W = \begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -0.84 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Hence

$$T = MW = \begin{bmatrix} 0.16 & 1 & 1 \\ -0.16 & 1 & 1 \\ -0.16 & 0.68 & 1 \end{bmatrix}$$

and

$$T^{-1} = \begin{bmatrix} 3.125 & -3.125 & 0 \\ 0 & 3.125 & -3.125 \\ 0.5 & -2.625 & 3.125 \end{bmatrix}$$

Thus

$$K = \begin{bmatrix} -0.16 & 0.84 & 0 \end{bmatrix} \begin{bmatrix} 3.125 & -3.125 & 0 \\ 0 & 3.125 & -3.125 \\ 0.5 & -2.625 & 3.125 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 3.125 & -2.625 \end{bmatrix}$$

4) B-6-16 consider the system defined by:

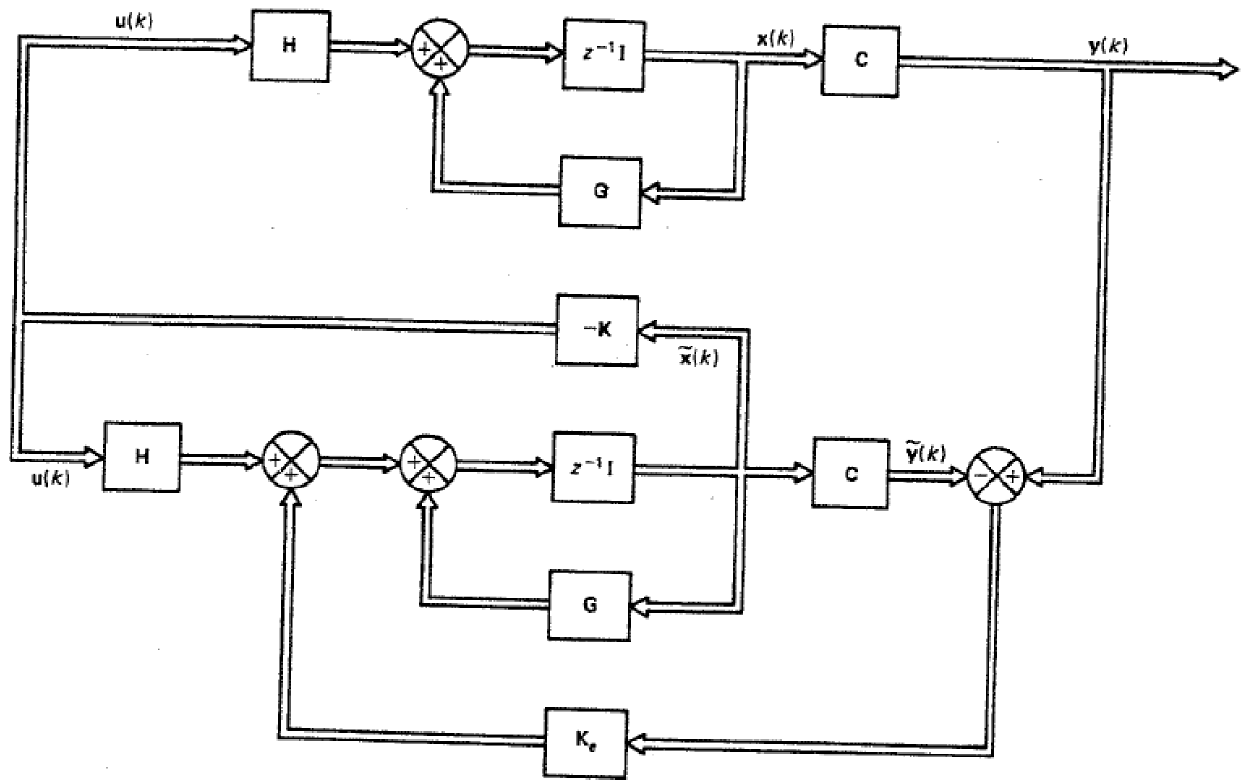
$$x((k+1)) = Gx(k) + Hu(k)$$
$$y(k) = Cx(k)$$

Where $G = \begin{bmatrix} 0 & -0.16 \\ 1 & -1 \end{bmatrix}$, $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$,

Using MATLAB, determine the observer feedback gain matrix K_e such that the desired eigenvalues for the observer matrix are

$u_1 = 0.5 + j0.5$, $u_2 = 0.5 - j0.5$

Assume that the system consifuration is identical to that shown in figure. Using Ackermann's formula, write a MATLAB program.



Observed state feedback control system

MATLAB Program for Problem B-6-16

```
% ---------- Design of state observer ----------

% ***** This program determines state observer gain matrix Ke
% by use of Ackermann's formula *****

% ***** Enter matrices G and C *****

G = [0  -0.16;1  -1];
C = [0  1];

% ***** Enter the observability matrix N and check its rank *****

N = [C'  G'*C'];
rank(N)

ans =

    2

% ***** Since the rank of the observability matrix is 2, design
% of observer is possible *****

% ***** Enter the desired characteristic polynomial by defining
% the following matrix J and entering statement poly(J) *****

J = [0.5+0.5*i    0
     0    0.5-0.5*i];

JJ = poly(J)

JJ =

   1.0000    -1.0000    0.5000

% ***** Enter characteristic polynomial Phi *****

Phi = polyvalm(poly(J),G);

% ***** The observer gain matrix Ke is obtained from *****

Ke = Phi*inv(N')*[0;1]

Ke =

    0.3400
   -2.0000
```