Department of Electrical and Computer Engineering

ECED4601 Digital Control System

Lab2 Conventional analysis and design methods of discrete-time system

1 Introduction

The objective of this lab is to familiarize the students with the analysis and design of discrete-time control system using the commands in MATLAB control toolbox. The conventional methods: Root-locus and bode-diagram methods are covered.

2 Root Locus

The root locus is a plot of the locations of all possible closed loop poles with proportional gain k and unity feedback. Let's take a look at the system in Fig. 1.
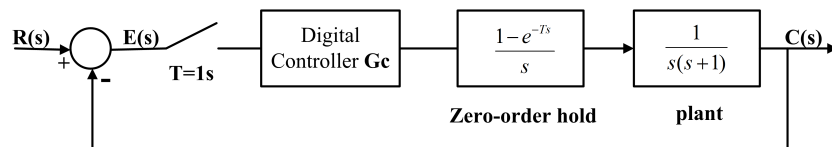

Figure 1: Discrete System Block Diagram

Let T=1. To plot the root locus for this system in MATLAB, open up a new m-file in MATLAB and type the following commands:

```
1  close all; clear all;
2  T=1;
3  Gpnum=[1];Gpden=[1 1 0];
4  Gps=tf(Gpnum,Gpden);%open-loop transfer function
5  Gz=c2d(Gps,T);%convert to discrete-time
6  figure(1);rlocus(Gz);axis([-3 1.5 -1.5 1.5]);
```

The rlocus(Gz) command plots the root locus for the system Gz. We can use the command zgrid(Zeta,Wn) to plot lines of constant damping ratio(ζ) and natural frequency(ω_n) to show the acceptable regions for certain design requirements. Say we need an overshoot less than 5% (which means a damping ratio ζ of greater than 0.7) and a settling time less than 15 second (which means a natural frequency $\omega_n = \frac{4}{\zeta T_s}$ greater than 0.38 rad/sec = 0.38*T rad/sample = 0.38 rad/sample). Note that, in the equation $T_s = \frac{4}{\zeta \omega_n}$, the unit of ω_n is rad/sec, but when you use the zgrid command, the Wn must be in the unit of rad/sample.

Add the following commands to the end of your m-file.

```
1  zeta = 0.7;
2  wn=0.38;
3  zgrid(zeta,wn);
```
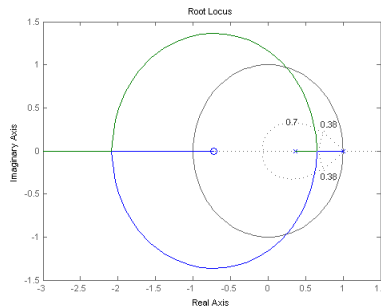

Figure 2: Root locus

From the plot in Fig. 2, to make the overshoot less than 5%, the poles have

to be inside the circle of $\zeta = 0.7$, and to make the settling time shorter than 15 second, the poles have to be located to the left of the semicircle $\omega_n = 0.38$. Left-Click on the root locus shows some of the system characteristics associated with that point.

To pick the poles in the desired region on the locus, use the command *rlocfind*.

```
1  [k,poles] = rlocfind(Gz)
```

On the root locus plot, click on the point where you want the closed-loop pole to be. Although you will be selecting only one pole, sometimes you get more than one pole, MATLAB will choose the other poles for you(selected poles are denoted with '+' sign). Don't forget to go back to the plot and check the location of all the poles because the response of the system will be affected by the other poles as well.

k is the gain that will result in the chosen poles which are given in 'poles'. Plot the step response of the system with this command.

```
1  sys_cl=feedback(k*Gz,1);
2  figure(2);step(sys_cl);
3  title(['Step response with gain ',num2str(k)]);
```
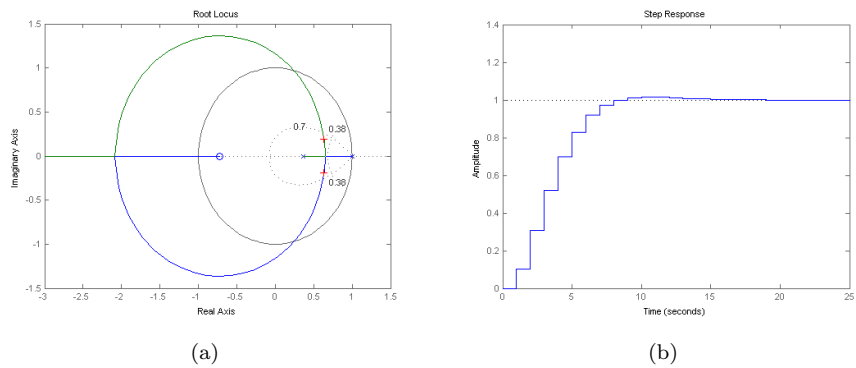

(a)   (b)

Figure 3: Step response with selected poles

We can see from fig. 3 that, the system response are as expected.

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Questions: For the same system what we discussed above, change the sampling period T, to T=0.5s and T=0.1s. Plot the root locus for each case. And from these plots you get, find the range of k for stability using the information on the root locus. What impact does the sampling period have on the system stability?
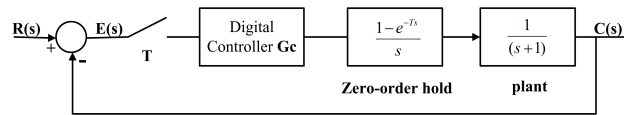
3   Root locus design

Figure 4: A first order system

Consider the system shown in Fig. 4. The design critiria for this control system is given as:

- settling time shorter than 5 seconds

- overshoot less than 10%

- zero steady-state error

Let T=0.5s, for the system without a controller Gz

1. use MATLAB to obtain the discrete-time transfer function Gp of the plant.

2. what's the system type?(type 0, 1, 2)

3. calculate the error constant for this type of system

4. is there any steady-state error for a unit-step input? if yes, calculate the value

5. plot the step response of the system

Use a PI controller(acts as a phase-lag compensator), given in this form

$$G_c(z) = K_p + \frac{K_I T_Z}{z-1} = \frac{(K_p + K_I T)z - K_p}{z-1} = \frac{K_d(z - z_0)}{z-1}$$

, where

$$K_d = K_p + K_I T, z_0 = \frac{K_p}{K_p + K_I T} = \frac{K_p}{K_d}$$

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What system type does a PI controller has?(type 0,1,2)(Hint: think about the continous-time function). How this will affect the steady-state error? Obtain open-loop function of the compensated system $Gc(z)Gp(z)$.

We need to determine the parameters K_p,K_I, to meet the design requirments.

The zero location will be picked first. For a phase-lag compensator, the zero is usually chosen close to the pole. After chosing the zero, we use root locus to find the desired poles and the gain, plot the step response of the compensated system to see if the design requirements have been achieved. If not, start with another zero and do it again, which basically is a trial-and-error process. Here we choose $z_0 = 0.55$.

1. What's the desired damping ratio ζ and natural frequency ω_n according to the design criteria? (Use the equation $P.O. = 100e^{(-\zeta\pi/\sqrt{(1-\zeta^2)})}$, or look up the plot of this relation)

2. Plot the root locus with the desired region for ζ and ω_n

3. On the root locus plot, pick a pole which makes the system meet the design requirements, plot the step response of the compensated system.
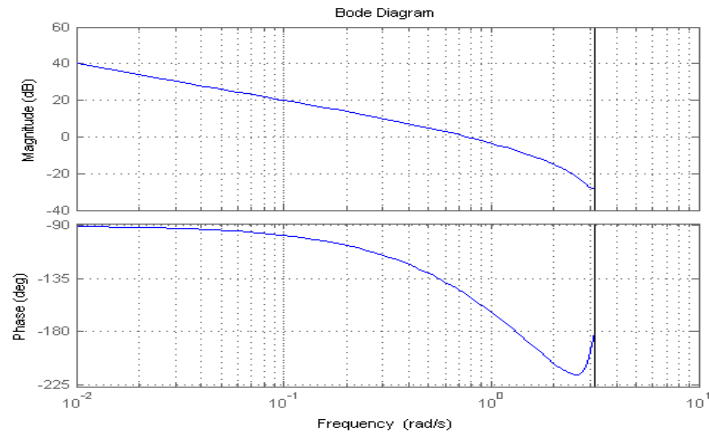
4 Bode Diagram

Consider the system in Fig.1. Type these commands in a new m-file and run it.
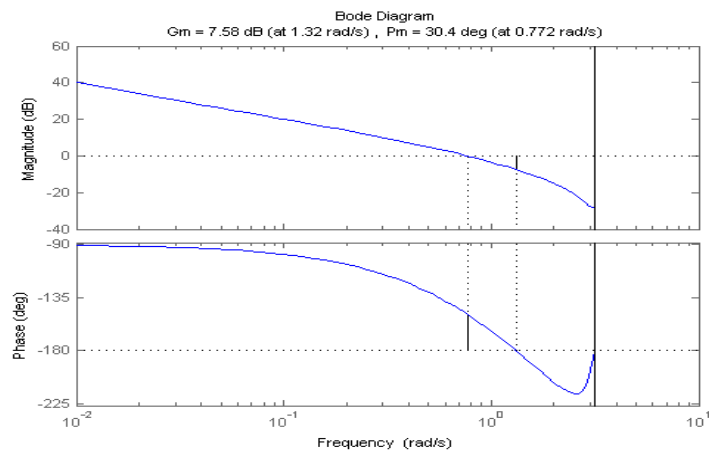
```
1  close all; clear all;
2  T=1;
3  Gpnum=[1];Gpden=[1 1 0];
4  Gps=tf(Gpnum,Gpden);
5  Gz=c2d(Gps,T);% open—loop transfer function
6  figure(1);bode(Gz);grid;
7  figure(2);margin(Gz);
```

The *bode* command gives the Bode plot for a transfer function. Using the *margin* command, we can obtain the gain and phase margins for a system directly. The bode plots for the open-loop system are shown in Fig. 5.

(a)

(b)

Figure 5: Bode Diagram

If we want to get the magnitude and phase value at a specific frequency, say 1 rad/sec, the following command can be used.

```
1   [mag,phase,w] = bode(Gz,1);%mag is in absolute units
```

There are two system characteristics we are interested in, overshoot $P.O$ which is related to ζ, and settlling time T_s. We can use these equations to

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establish relationship between frequency domain specs and time domain specs.

$$PM = 100\zeta, \ \ \zeta \leq 0.7$$

,where PM is the phase margin.

$$\omega_B = \frac{4}{T_s\zeta}\sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

, where ω_B is the bandwidth frequency, corresponding to a gain of -3 dB in the **CLOSED-LOOP** bode plot.

Assume the design sepcs are:

- settling time shorter than 15 seconds

- overshoot less than 5%

1. What's the desired phase margin and bandwidth frequency?

2. What's the bandwidth ω_B of this system?(closed-loop bode plot is needed)

3. We can use a PD controller to increase the phase margin. The transfer function of PD controller can be expressed as $G_c(z) = k(z - z_0)/z$, where k and z_0 need to be determined. Let $z_0 = 0.5$, change k to meet the design specs. Find the appropriate value of k, plot the open-loop bode plot and step response of the compensated closed-loop system. (Hint: phase margin can be read off the open-loop bode plot, bandwidth can be read off the closed-loop bode plot)

5 Guidelines for writing the Lab Report

The report has to include:

- **Title page** - Including: course number, lab title, student names, student IDs, Date the report was submitted.

- **Abstract** - A summary of the contents of the lab report.

- **Procedure of methods or approach to the design or/and conduct of the experiments**

- **Diagrams** - Include all Simulink blocks used in the lab if any.

- **Plots** - All plots of system response should be included in your report, including title, labels with unit, legends, etc. Note that Diagrams and Plots must be computer generated. Hand-drawn plots will not be accepted. All diagrams and plots must be labelled. The labels and annotations can be done by hand if appropriate but have to be clear.

- **Code** - Include any code used in the lab. The code must be commented properly. It is good practice to put your code in separate m-files.

- **Discussion and answer to questions if any**.

- **Conclusion or any other relevant ideas**.