IV.1.2 Cascade Compensation Networks

The compensation network may be chosen as follows:

$$G_c(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)} \qquad 4.1$$

We shall study the first order compensator as the high order compensator can be implemented by cascading several first order compensators.

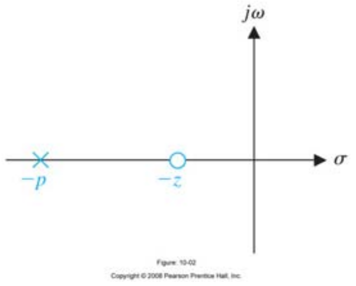
A first order compensator:

$$G_c(s) = \frac{K(s+z)}{(s+p)} \qquad 4.2$$

Goal: to select proper values of K, z, and p to provide a desired performance.

Phase-lead network: $|z| < |p|$, **Phase-lag network:** $|z| > |p|$,

a) Phase lead network:



Special case for phase lead network: $|z| << |p|, and\ z \to 0$, $G_c(s) = \frac{Ks}{(s+p)} \approx \frac{Ks}{p}$, it is a differentiator with the phase angle of 90^0.

In general:

$$G_c(s) = \frac{K(s+z)}{(s+p)} \to G_c(jw) = \frac{K(jw+z)}{(jw+p)} = \frac{Kz}{p}\frac{\left(j\frac{w}{z}+1\right)}{\left(j\frac{w}{p}+1\right)} = K_1\frac{1+jw\alpha\tau}{1+jw\tau} \qquad 4.3$$

Where $\tau = \frac{1}{p}, \alpha = \frac{p}{z}, K_1 = \frac{K}{\alpha}$

The frequency response is shown in figure with

$$\Phi(w) = \tan^{-1}(\alpha w\tau) - \tan^{-1}(w\tau) \qquad 4.4$$

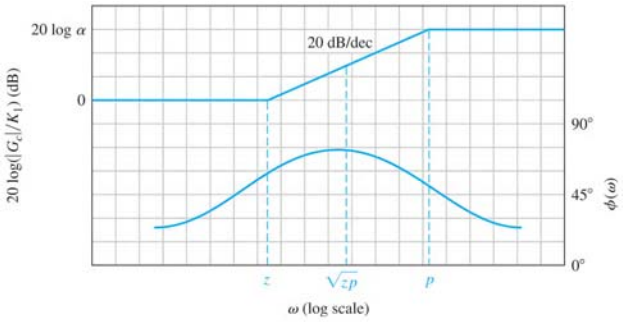


Figure: 10-03

The maximum value of the phase lead occurs a frequency w_m, where w_m is the geometric mean of $p = \dfrac{1}{\tau}$ *and* $z = \dfrac{1}{\alpha\tau}$,

Equation 4.4 can be rewritten as $\Phi = \tan^{-1}\left(\dfrac{\alpha w\tau - w\tau}{1+(w\tau)^2\alpha}\right)$ 4.5

To determine the maximum phase lead angle, let $\dfrac{\partial\Phi}{\partial w} = \dfrac{\partial\left(\tan^{-1}\left(\dfrac{\alpha w\tau - w\tau}{1+(w\tau)^2\alpha}\right)\right)}{\partial w} = 0$, then we

Will have $w_m = \sqrt{zp} = \sqrt{\dfrac{1}{\tau}\dfrac{1}{\alpha\tau}} = \dfrac{1}{\tau\sqrt{\alpha}}$ 4.6

Then substituting frequency for the maximum phase angle $w_m = \dfrac{1}{\tau\sqrt{\alpha}}$, we have

$$\tan\Phi_m = \left.\left(\frac{\alpha w\tau - w\tau}{1+(w\tau)^2\alpha}\right)\right|_{w=\frac{1}{\tau\sqrt{\alpha}}} = \frac{\alpha/\sqrt{\alpha} - 1/\sqrt{\alpha}}{1+1} = \frac{\alpha - 1}{2\sqrt{\alpha}} \Rightarrow \sin\Phi_m = \frac{\alpha-1}{\alpha+1} \qquad 4.7$$

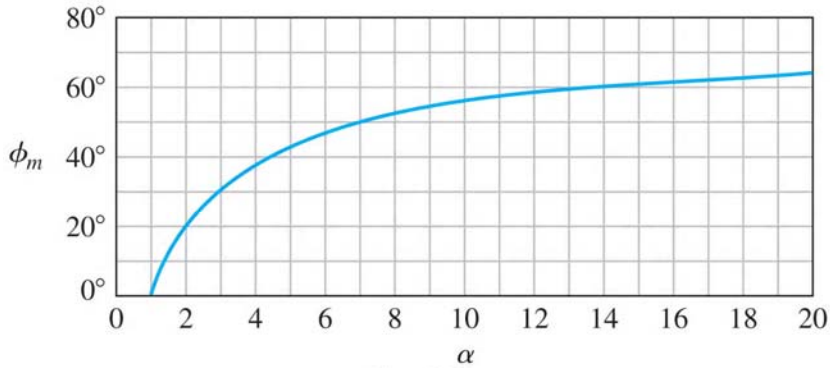


Figure: 10-05

b) Phase lag network:

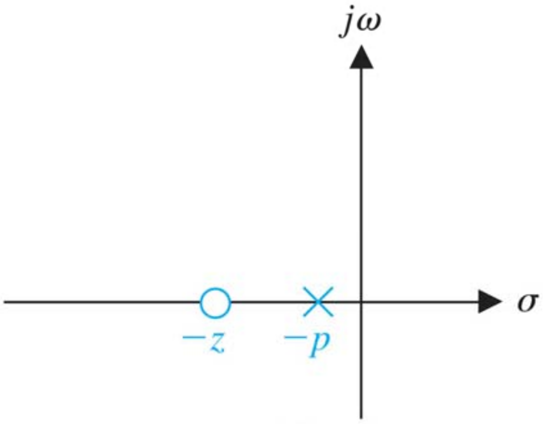


Figure: 10-07

Special case for phase lag network: $|z| >> |p|, and\ p \to 0$, $G_c(s) = \frac{K(s+z)}{(s+p)} \approx \frac{Kz}{s}$, it is a integrator with the phase angle of -90^0.

In general:

$$G_c(s) = \frac{K(s+z)}{(s+p)} \to G_c(jw) = \frac{K(jw+z)}{(jw+p)} = \frac{Kz}{p}\frac{\left(j\frac{w}{z}+1\right)}{\left(j\frac{w}{p}+1\right)} = K_1\frac{1+jw\tau}{1+jw\alpha\tau} \qquad 4.8$$

Where $\tau = \frac{1}{z}, \alpha = \frac{z}{p}, K_1 = K\alpha$

The frequency response is shown in figure with

$$\Phi(w) = \tan^{-1}(w\tau) - \tan^{-1}(w\alpha\tau) \qquad 4.9$$

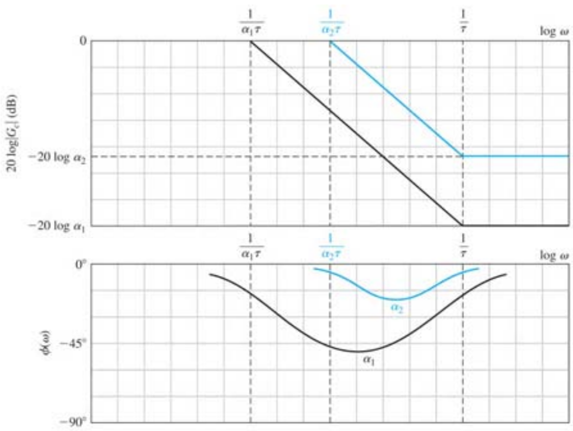


Figure: 10-08

The maximum value of the phase lag occurs at frequency w_m, where w_m is the geometric mean of $z=\frac{1}{\tau}$ *and* $p=\frac{1}{\alpha\tau}$,

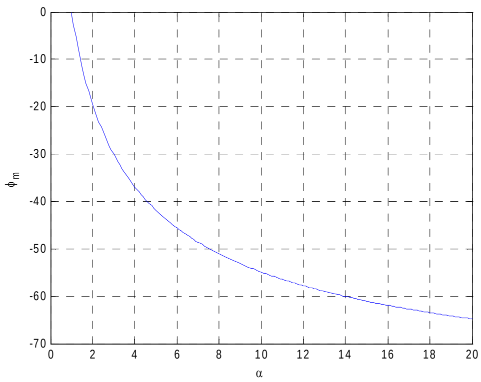
Equation 4.9 can be rewritten as $\Phi=\tan^{-1}\left(\frac{w\tau - w\alpha\tau}{1+(w\tau)^2\alpha}\right)$ 4.10

To determine the maximum phase lead angle, let $\frac{\partial\Phi}{\partial w}=\frac{\partial\left(\tan^{-1}\left(\frac{w\tau-w\alpha\tau}{1+(w\tau)^2\alpha}\right)\right)}{\partial w}=0$, then we

Will have $w_m=\sqrt{zp}=\sqrt{\frac{1}{\tau}\frac{1}{\alpha\tau}}=\frac{1}{\tau\sqrt{\alpha}}$ 4.11

Then substituting frequency for the maximum phase angle $w_m=\frac{1}{\tau\sqrt{\alpha}}$, we have

$$\tan\Phi_m=\left(\frac{w\tau-w\alpha\tau}{1+(w\tau)^2\alpha}\right)\Bigg|_{w=\frac{1}{\tau\sqrt{\alpha}}}=\frac{1/\sqrt{\alpha}-\alpha/\sqrt{\alpha}}{1+1}=\frac{1-\alpha}{2\sqrt{\alpha}}\Rightarrow\sin\Phi_m=\frac{1-\alpha}{\alpha+1} \quad 4.12$$



Examples of phase-lead or phase-lag compensators:

1) phase lead :

$$Gc=\frac{R_4R_2}{R_3R_1}(R_1C_1s+1)$$

IV.3 Phase-lead design (bode diagram and root locus)

Phase-lead design basically adds the phase to the uncompensated network in order to increase the phase margin to result a stable compensated system.

Phase lead design steps using bode diagram:

1) Evaluate the uncompensated system phase margin when the error constants are satisfied.
2) Allowing for a small amount of safety, determine the necessary additional phase lead
3) Evaluate α from equation 4.7
4) Evaluate the $10\log\alpha$ and determine the frequency where the uncompensated magnitude curve is equal to $-10\log\alpha$. This frequency is the new 0-dB crossover frequency and w_m simultaneously.
5) Calculate the pole and zero, $p = \frac{1}{\tau} = w_m\sqrt{\alpha}$ *and* $z = \frac{1}{\alpha\tau} = \frac{p}{\alpha}$
6) Draw the compensated frequency response. Check to validate the design. Finally, raise the gain of the amplifier to account for the attenuation $\frac{1}{\alpha}$

Example 4.2 consider a unity feedback system with the transfer function

$G(s) = \frac{K}{s(s+2)(s+4)}$, we desire to obtain the dominate roots with $w_n = 3$ and $\xi = 0.5$. We wish to obtain a $K_v = 2.7$. Show that we require a compensator

$G_c(s) = \frac{7.53(S+2.2)}{(s+16.4)}$, determine the value of K that should be selected.

Phase margin requirement $\phi_{PM} = \frac{\xi}{0.01} = 50^0$, $K = 2*4*kv = 2*4*2.7 = 21.6$,let K=22

1) PM of $G(s) = \frac{22}{s(s+2)(s+4)}$ is 22.7^0

2) Phase lead $\phi_m = 50 - 22.65 = 27.35^0 \rightarrow 50^0$

3) $\sin\Phi_m = \frac{\alpha - 1}{\alpha + 1} \Rightarrow \alpha = 7.45$

4) $10\log 10(\alpha) = -4.77 \Rightarrow w_m = 6$

5) $p = \frac{1}{\tau} = 6\sqrt{7.45} = 6*\sqrt{7.45} = 16.4$ *and* $z = \frac{1}{\alpha\tau} = \frac{p}{\alpha} = \frac{16.4}{7.45} = 2.2$

$$G_c(s) = \frac{7.53(S+2.2)}{(s+16.4)}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\frac{R(s)}{1+G_c(s)G(s)H(s)}$$

The steady state error of a system depends on the number of poles at the origin for $L(s) = G_c(s)G(s)H(s)$.

If the steady state accuracy is not sufficient, we will introduce an integration-type network $G_c(s)$ in order to compensate for the lack of integration in the uncompensated loop transfer function.

Popular PI controller is defined as $G_c(s) = K_p + \frac{K_I}{s}$

Example 4.4: find out the e_{ss} for type 0 system with and without PI controller applied.

$$G(s) = \frac{k\prod_{i=1}^{M}(s+z_i)}{\prod_{k=1}^{Q}(s+p_k)}$$

a) Without the PI controller, assume $H(s)=1$

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G(s)}\frac{A}{s} = \frac{A}{1+\lim_{s\to 0} G(s)} = \frac{A}{1+G(0)} = \frac{A}{1+k\frac{\prod_{i=1}^{M} z_i}{\prod_{k=1}^{Q} p_k}}$$

b) With the PI controller, assume $H(s)=1$

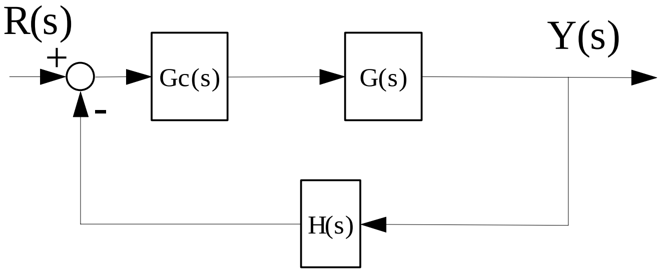
$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+G_c(s)G(s)}\frac{A}{s} = \lim_{s\to 0}\frac{A}{1+(K_p+\frac{K_I}{s})G(s)} = \lim_{s\to 0}\frac{As}{s+K_p s+K_I G(0)} = 0$$

Transient performance can be adjusted to satisfy the system specification by adjusting K, Kp, KI.

Phase lag design steps using bode diagram:

1) Evaluate the uncompensated system phase margin when the error constants are satisfied.
2) Determine the phase margin of the uncompensated system and, if it is insufficient, proceed with the following steps
3) Determine the frequency where the phase margin requirement would be satisfied if the magnitude curve crossed the 0dB line. $w_c^{'}$. Allow for 5 degree lag.
4) Place the zero of the compensator one decade below the new crossover frequency $w_c^{'}$
5) Measure the necessary attenuation at $w_c^{'}$ to ensure that the magnitude curve crosses at this frequency.
6) Calculate α by noting that the attenuation introduced by the phase-lag network is $-20\log\alpha$ at $w_c^{'}$.
7) Calculate the pole as $w_p = \frac{1}{\alpha\tau} = \frac{w_z}{\alpha}$, and the design is completed.

Example 4.5 (10.14)a Robot will be operated by NASA to build a permanent lunar station. The position control system for the gripper tool is shown in figure, where H(s)=1, and



$G(s) = \dfrac{5}{s(s+1)(0.25s+1)}$, determine a compensator that will provide a phase margin of 45^0.

1) Evaluate the uncompensated system phase margin when the error constants are satisfied. Phase margin is 0.
2) Determine the phase margin of the uncompensated system and. It is insufficient, proceed with the following steps
3) Determine the frequency where the phase margin requirement would be satisfied if the magnitude curve crossed the 0dB line. $w_c^{'}$. Allow for 5 degree lag
   Add 5 degree to 45 degree as the phase margin. So phase margin is 50 degree.
   Find the frequency 0.598 at which phase margin is 50 degree so $w_c^{'} = 0.598$
4) Place the zero of the compensator one decade below the new crossover frequency $w_c^{'}$
   $w_z = w_c^{'}/10 = 0.0598$
5) Measure the necessary attenuation at $w_c^{'}$ to ensure that the magnitude curve crosses at this frequency.
   The attenuation is 16.9 dB
6) Calculate α by noting that the attenuation introduced by the phase-lag network is $-20\log\alpha$ at $w_c^{'}$.

$20\log\alpha = 16.9dB \rightarrow \alpha = 7$

7) Calculate the pole as $w_p = \dfrac{1}{\alpha\tau} = \dfrac{w_z}{\alpha}$, and the design is completed.

$$w_p = \frac{1}{\alpha\tau} = \frac{w_z}{\alpha} = 0.0598/7$$

$$G_c(s) = \frac{(s+0.0598)}{7(s+0.0598/7)}$$