Ref: "Introduction to Robotics" "John J. Craig
Addison-wesley Publishing Company

Notations

- upper case : Vectors and matrices. Lower case : scalars
- leading subscripts and superscripts : identify which coordinate system a quantity is written in

Example : $^{A}P \rightarrow$ Position vector P in $\{A\}$ $\{A\}, \{B\}$,

$^{A}_{B}R \rightarrow$ rotation matrix R : specifies the relation between Coordinates

- Trailing ~~sub~~ super scripts are used for indicating the inverse or tranpose of a matrix

$$R^{-1}, \quad R^{T}$$

- Trailing subscripts are not subject to strict convention, but may indicate a component or a specific vector or matrix. P_x. P_y. P_z. P_{bolt}

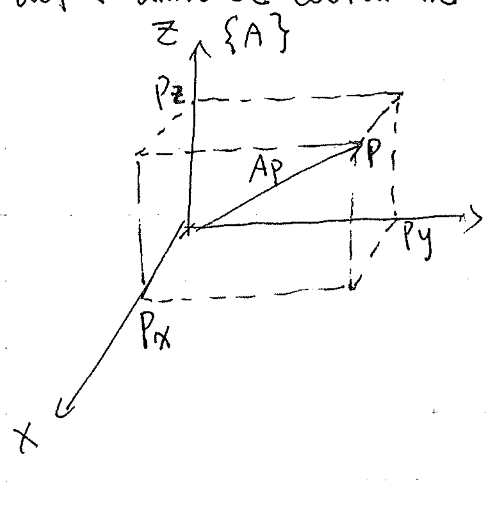
Trigonometrix function $\cos\theta_1 = C\theta_1 = C_1$ $\sin(\theta_1+\theta_2) = S\theta_{12} = S_{12}$

Vector sum $^{6}W_4 \neq {}^{0}W_1 + {}^{1}W_2 + {}^{2}W_3 + {}^{3}W_4$

$$^{6}W_4 = {}^{6}W_1 + {}^{6}W_2 + {}^{6}W_3 + {}^{6}W_4$$

Remarks : Don't sum vectors unless they are in the same Coordinate systems

def : universe coordinate system : A Coordinate system to which everything can be referenced.



Position $\rightarrow P$

3 D vector $\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = {}^{A}P$

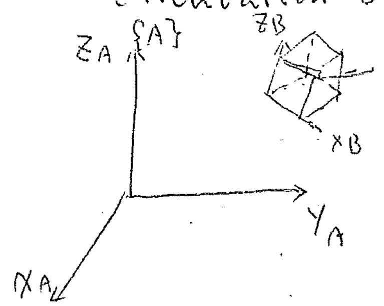
where

P_x = the projection of ^{A}P onto X_A axis

P_y = the projection of ^{A}P onto Y_A axis

P_z = the projection of ^{A}P onto Z_A axis

Orientation of a 3-D body: Rotation Matrix.



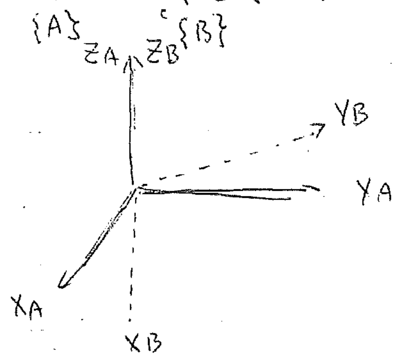
object in frame {A}

attach a coordinate system to the body {B}.

then the orientation of the body can be described by frame {B}. i.e. by the three unit vectors along its ~~prin~~ principal axes: ${}^{A}\hat{X}_B$ – ${}^{A}\hat{Y}_B$ ${}^{A}\hat{Z}_B$

$\Rightarrow$ rotation matrix ${}^{A}_{B}R \triangleq [\,{}^{A}\hat{X}_B \;\; {}^{A}\hat{Y}_B \;\; {}^{A}\hat{Z}_B\,]$

example: {B} is obtained by rotating {A} about Z_A by 30°. Find ${}^{A}_{B}R$



$${}^{A}\hat{Z}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad {}^{A}\hat{Y}_B = \begin{bmatrix} -1/2 \\ \sqrt{3}/2 \\ 0 \end{bmatrix} \quad {}^{A}\hat{X}_B = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{bmatrix}$$

$$\text{So } {}^{A}_{B}R = [\,{}^{A}\hat{X}_B \;\; {}^{A}\hat{Y}_B \;\; {}^{A}\hat{Z}_B\,] = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

recall:
$$ {}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} {}^{A}\hat{X}_B \cdot {}^{A}\hat{X}_A & {}^{A}\hat{Y}_B \cdot {}^{A}\hat{X}_A & {}^{A}\hat{Z}_B \cdot {}^{A}\hat{X}_A \\ {}^{A}\hat{X}_B \cdot {}^{A}\hat{Y}_A & {}^{A}\hat{Y}_B \cdot {}^{A}\hat{Y}_A & {}^{A}\hat{Z}_B \cdot {}^{A}\hat{Y}_A \\ {}^{A}\hat{X}_B \cdot {}^{A}\hat{Z}_A & {}^{A}\hat{Y}_B \cdot {}^{A}\hat{Z}_A & {}^{A}\hat{Z}_B \cdot {}^{A}\hat{Z}_A \end{bmatrix}$$

$$= \begin{bmatrix} {}^{B}\hat{X}_B \cdot {}^{B}\hat{X}_A & {}^{B}\hat{Y}_B \cdot {}^{B}\hat{X}_A & {}^{B}\hat{Z}_B \cdot {}^{B}\hat{X}_A \\ {}^{B}\hat{X}_B \cdot {}^{B}\hat{Y}_A & {}^{B}\hat{Y}_B \cdot {}^{B}\hat{Y}_A & {}^{B}\hat{Z}_B \cdot {}^{B}\hat{Y}_A \\ {}^{B}\hat{X}_B \cdot {}^{B}\hat{Z}_A & {}^{B}\hat{Y}_B \cdot {}^{B}\hat{Z}_A & {}^{B}\hat{Z}_B \cdot {}^{B}\hat{Z}_A \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{bmatrix}$$

$$= [\,{}^{B}\hat{X}_A \;\; {}^{B}\hat{Y}_A \;\; {}^{B}\hat{Z}_A\,]^T = {}^{B}_{A}R^T$$

$${}^{A}_{B}R^T \, {}^{A}_{B}R = ? \quad \begin{bmatrix} {}^{A}\hat{X}_B \\ {}^{A}\hat{Y}_B \\ {}^{A}\hat{Z}_B \end{bmatrix} [\,{}^{A}\hat{X}_B \;\; {}^{A}\hat{Y}_B \;\; {}^{A}\hat{Z}_B\,] = I_3 \;\Rightarrow\; {}^{A}_{B}R^{-1} = {}^{A}_{B}R^T$$

$$\Rightarrow {}^{A}_{B}R = {}^{B}_{A}R^T = {}^{B}_{A}R^{-1}$$

Description of a Frame :

A frame {B} is characterized by the ~~Des~~ position of its origin and the orientation of the frame relative to a reference frame {A}

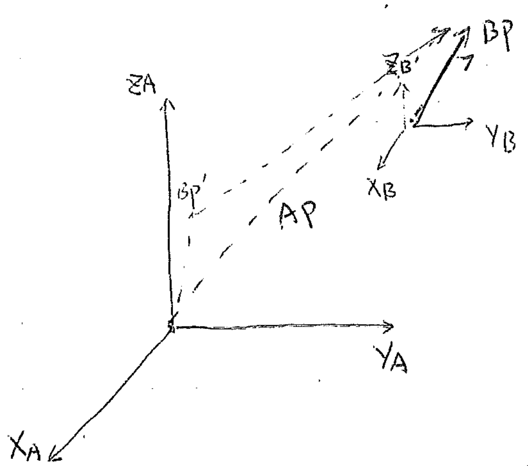
$$\{B\} = \{\ {}^{A}_{B}R \quad {}^{A}P_{BORG}\ \}.$$

Two Special Case : ① ${}^{A}_{B}R = I$ {B} represents a position

② ${}^{A}P_{BORG} = 0$. {B} represents an orientation.

Changing description from Frame to Frame .

i. mapping involving translated frames

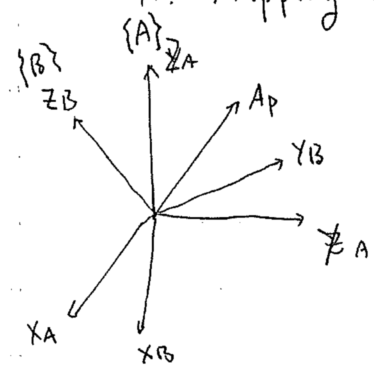


$$Ap \ ? \quad Bp.$$

$$Ap = Bp' + Ap_{BORG}$$

$$= Bp + Ap_{BORG}$$

ii. mapping involving rotated frames



$$Ap \ ? \quad Bp.$$

$$Ap = \begin{bmatrix} Ap_x \\ Ap_y \\ Ap_z \end{bmatrix} = \begin{bmatrix} {}^{A}X_A^T \cdot Ap \\ {}^{A}Y_A^T \cdot Ap \\ {}^{A}Z_A^T \cdot Ap \end{bmatrix} = \begin{bmatrix} {}^{B}X_A^T \cdot Bp \\ {}^{B}Y_A^T \cdot Bp \\ {}^{B}Z_A^T \cdot Bp \end{bmatrix}$$

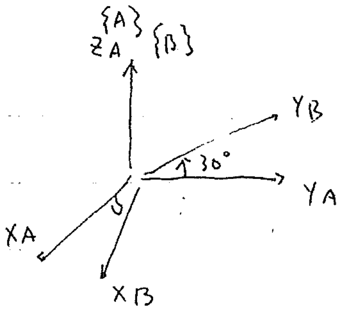
$$Q_1^T Q_2 = Q_2^T Q_1 = |Q_1|\,|Q_2| \cos(\hat{\theta}_1, \hat{\theta}_2)$$

$\Rightarrow Q_1^T Q_2$ is scalar

$\Rightarrow$ projection is frame independent!

$$\text{So } Ap = \begin{bmatrix} {}^{B}X_A^T \\ {}^{B}Y_A^T \\ {}^{B}Z_A^T \end{bmatrix} \cdot Bp = {}^{B}_{A}R^T \cdot Bp = {}^{A}_{B}R \cdot Bp$$

Example: $\{B\}$ is obtained by rotating $\{A\}$ about Z_A axis by $30°$.



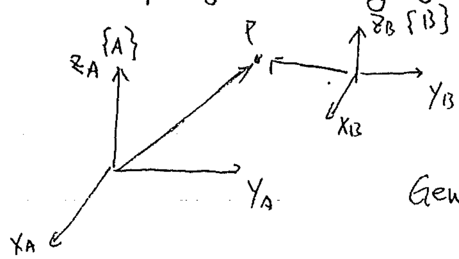
Given $^{B}P \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ find ^{A}P

Solution: $^{A}\hat{X}_B = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{bmatrix}$ $\quad ^{A}\hat{Y}_B = \begin{bmatrix} -1/2 \\ \sqrt{3}/2 \\ 0 \end{bmatrix}$ $\quad ^{A}\hat{Z}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$^{A}_{B}R = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^{A}P = {}^{A}_{B}R\,{}^{B}P = \begin{bmatrix} \sqrt{3} & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

iii mapping involving general frames:



Translation only $^{A}P = {}^{B}P + {}^{A}P_{BORG}$

Rotation only: $^{A}P = {}^{A}_{B}R\,{}^{B}P$

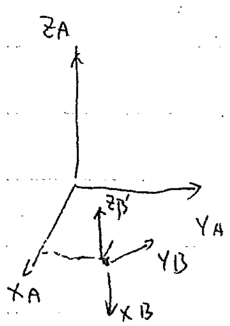
General: $^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$

$$\Longleftrightarrow \begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} ^{A}_{B}R & ^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = {}^{A}_{B}T \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

$$^{A}_{B}T = \begin{bmatrix} ^{A}_{B}R & ^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix}_{4\times 4}$$ is called homogeneous transform

Example: $\{B\}$ is obtained by rotating $\{A\}$ about Z_A axis by $30°$ and translating 10 units in X_A direction and 5 units in Y_A direction.



Given $^{B}P = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ find ^{A}P

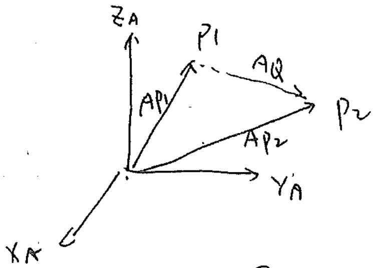
$$^{A}_{B}R = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad ^{A}P_{BORG} = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$$

$$^{A}_{B}T = \begin{bmatrix} ^{A}_{B}R & ^{A}P_{BORG} \\ 0\,0\,0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 10 \\ 1/2 & \sqrt{3}/2 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = {}^{A}_{B}T \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} 9.36 \\ 8.09 \\ 5.0 \\ \cdots \end{bmatrix} \Rightarrow {}^{A}P = \begin{bmatrix} 9.36 \\ 8.09 \\ \cdots \end{bmatrix}$$

operators : Translations, Rotations.

i) Translation operators.



Given frame $\{A\}$, we want to move point P_1 to P_2

$$AP_2 = AP_1 + AQ.$$

Moving P_1 to P_2 is equivalent to adding vector AQ to AP_1

Since
$$\begin{bmatrix} AP_2 \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} I_3 & AQ \\ \hline 000 & 1 \end{array}\right] \begin{bmatrix} AP_1 \\ 1 \end{bmatrix}$$

translation can be accomplished by the matrix operator

$$\begin{bmatrix} AP_2 \\ 1 \end{bmatrix} = Trans(^A\hat{Q}, |Q|) \begin{bmatrix} AP_1 \\ 1 \end{bmatrix}, \text{ where } Trans(^A\hat{Q}, |Q|) = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Remark: Since $\overrightarrow{P_1P_2} = AQ$, moving a point from one location to another is the same as the transform describing a frame obtained by translating frame $\{A\}$ by AQ

ii) rotation operator

Given $\{A\}$, we rotate a vector AP_1 about a vector $\hat{K}$ by θ degrees and call the resulting vector AP_2, Denote the action as operator $RoT(\hat{K}, \theta)$.

$$\begin{bmatrix} AP_2 \\ 1 \end{bmatrix} = RoT(\hat{K}, \theta) \begin{bmatrix} AP_1 \\ 1 \end{bmatrix}, \text{ where } RoT(\hat{K}, \theta) = \begin{bmatrix} R & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 000 & 1 \end{bmatrix}$$

How to find R?

$$RoT(\hat{x}, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad RoT(\hat{Y}, \theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RoT(\hat{z}, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

prove by self

Remark: operator which rotates a vector by R is the same as the transform which describe a frame rotated by R relative to reference frame $\{A\}$

General operator that rotates and translates a vector

Given {A} and a vector ${}^A P_1$, let T be an operator that rotates the vector about a rotation axis by θ { R }, and translate it by a vector Q to get ${}^A P_2$

$$\begin{bmatrix} {}^A P_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} {}^A P_1 \\ 1 \end{bmatrix}, \quad \text{where } T = \begin{bmatrix} R & Q \\ 000 & 1 \end{bmatrix}$$

R first, T second $R+T = \begin{bmatrix} I & Q \\ 0 & 1 \end{bmatrix}\begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R & Q \\ 0 & 1 \end{bmatrix}$

wrong $T+R = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} I & Q \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R & RQ \\ 0 & 1 \end{bmatrix}$

Remark: T is same as the transformation which describes a frame rotated by R and translated by Q relative to the reference frame {A}

Given ${}^A_B T$ and ${}^B_C T$ then ${}^A_C T = {}^A_B T \, {}^B_C T$

inverse: $({}^A_B T)^{-1} = {}^B_A T$

$${}^B_A T = \begin{bmatrix} {}^A_B R & {}^A P_{BORG} \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} {}^B_A R & {}^B P_{AORG} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^A_B R^T & -{}^A_B R^T \, {}^A P_{BORG} \\ 0 & 1 \end{bmatrix}$$

? ${}^B P_{AORG} = -{}^A_B R^T \, {}^A P_{BORG}$

Since ${}^B({}^A P_{BORG}) = {}^B_A T \left[{}^A P_{BORG}\right] = \cancel{{}^B_A R \, {}^A P_{BORG} + {}^B P_{BORG}}$

$$= \begin{bmatrix} {}^B_A R & {}^B P_{AORG} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} {}^A P_{BORG} \\ 1 \end{bmatrix} = {}^B_A R \, {}^A P_{BORG} + {}^B P_{AORG} = 0$$

$$\Rightarrow {}^B P_{AORG} = -{}^B_A R \, {}^A P_{BORG} = -{}^A_B R^T \, {}^A P_{BORG}$$

Remark: ${}^B({}^A P_{BORG}) = 0$

Link parameters : Denavit-Hartenberg notation

Three steps to obtain link parameters using D-H notation.

1. Number the links and joint axes from base to tip

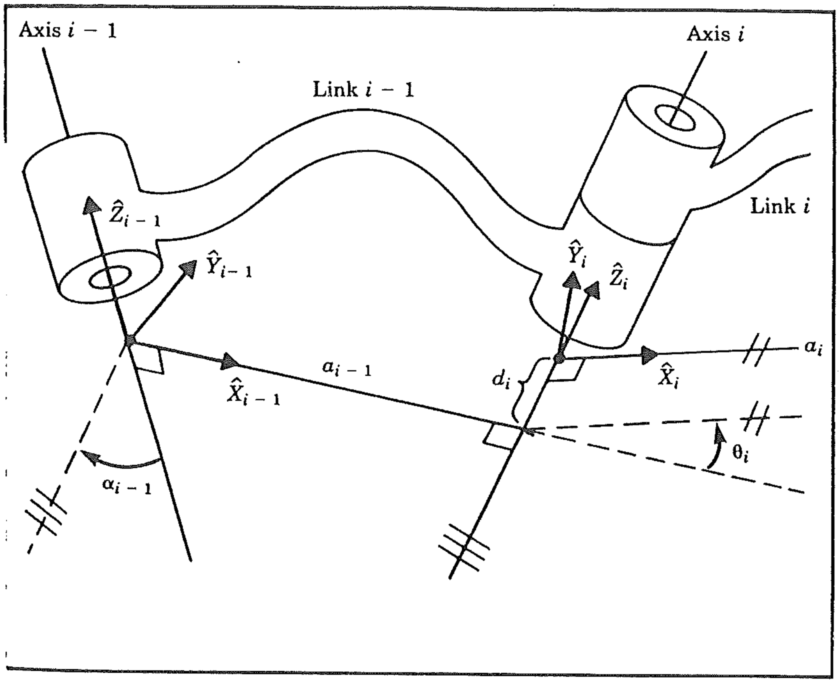


FIGURE 3.5 Link frames are attached so that frame {i} is attached rigidly to link i.

2. Attach link frames as follows
   Z_{i-1} axis coincides with joint axis $i-1$;
   X_{i-1} axis coincides with the common normal between joint axis $i-1$ and joint axis i
   Y_{i-1} axis can be obtained by using the right hand rule.

3. For each link, four parameters are assigned to describe the link itself and its connection,

For link i four parameters are as follows

α_{i-1} : link twist (angle from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured about along $\hat{X}_{i-1}$

a_{i-1} : link length (distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured by $\hat{X}_{i-1}$

d_i : link offset (distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i : joint angle (angle from $\hat{X}_{i-1}$ to $\hat{X}_i$) measured about $\hat{Z}_i$

Remarks: ① Each frame, {i} is attached rigidly to link (i), and moves with link;

② α_{i-1}, a_{i-1} always fixed : if robot is made

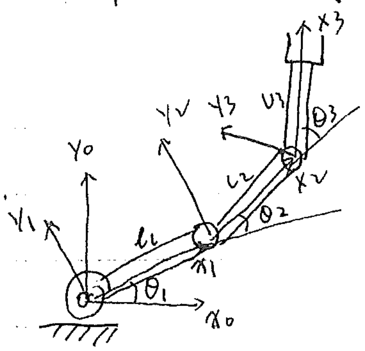
③ when joint i is rotational, the θ_i is a variable and d_i is fixed

when joint i is prismatic, then d_i is a variable and θ_i is fixed.

④ Frame {0} attached to the base is always fixed with its Z_0 axis.

aligned with joint axis1 $\Rightarrow$ $\alpha_0 = 0$, $a_0 = 0$

Example 3 d.o.f plannar manipulator, Find D-H table parameters.



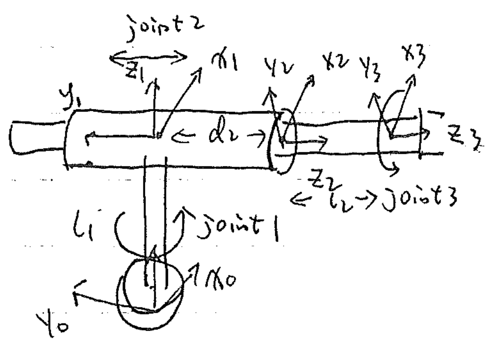
① set up frames {0} {1} {2} {3}

All Z-axes are going out. {0} and {1} have same origin

X axis are coincident with the corresponding links.

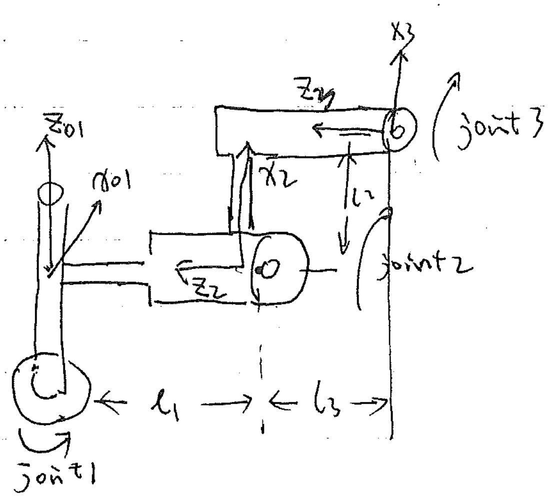
② Fill in the D-H table.

Link i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	θ_3



2R-1P non-plannar manipulator

Link i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	l_1	θ_1
2	$\pi/2$	0	d_2	0
3	0	0	l_2	θ_3



3R robot.

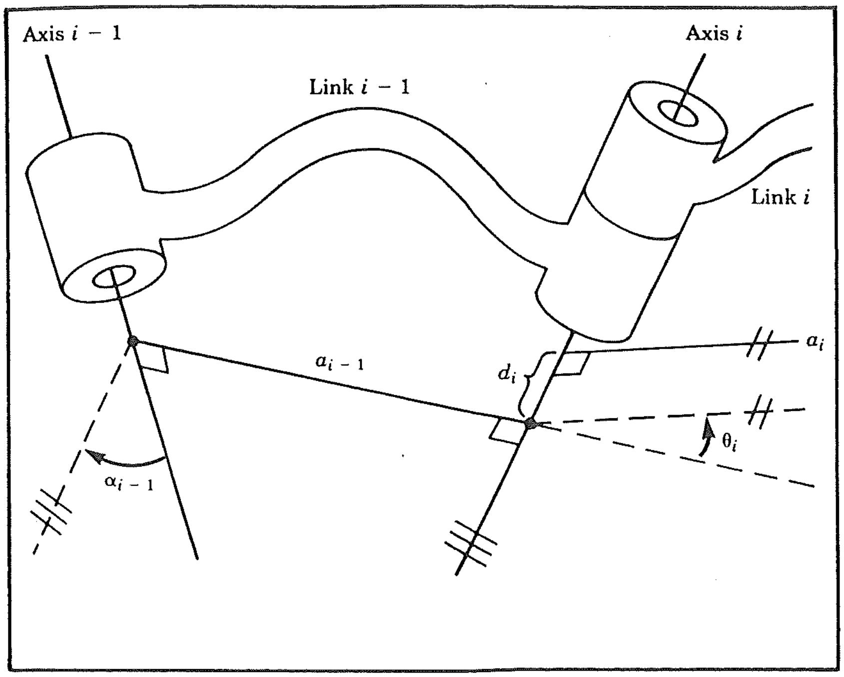
D-H table

Link i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\pi/2$	0	l_1	θ_2
3	0	l_2	$-l_3$	θ_3

Manipulator kinematics:

Let ${}^{i-1}_{i}T$ be the homogenous transformation associated with frame $\{i\}$ and $\{i-1\}$ that are attached to link i and $i-1$, then any position vector in frame $\{i\}$, ${}^{i}P$ is related to the same vector in $\{i-1\}$ by $\begin{bmatrix} {}^{i-1}P \\ 1 \end{bmatrix} = {}^{i-1}_{i}T \begin{bmatrix} {}^{i}P \\ 1 \end{bmatrix}$

How to compute ${}^{i-1}_{i}T$



To compute ${}^{i-1}_{i}T$, we introduce several intermediate frames between frame $\{i\}$ and $\{i-1\}$.

$\{i-1\}$

$\downarrow$ rotation about X_{i-1} by α_{i-1}

$\{R\}$

$\downarrow$ translation along X_{i-1} by a_{i-1}

$\{Q\}$

$\downarrow$

$\{P\}$ rotation about Z_i by θ_i

$\downarrow$ translation along Z_i by d_i

$\{i\}$

$$ {}^{i-1}_{i}T = {}^{i-1}_{R}T \cdot {}^{R}_{Q}T \cdot {}^{Q}_{P}T \; {}^{P}_{i}T $$

$$ = \mathrm{RoT}(X_{i-1}, \alpha_{i-1}) \cdot \mathrm{Trans}(X_{i-1}, a_{i-1}) \cdot \mathrm{RoT}(Z_i, \theta_i) \cdot \mathrm{Trans}(Z_i, d_i) $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

For n-Link robot : The Cartesian position and orientation of the ~~equation of an n-Link manipulator~~ last Link in a n-Link robot manipulator w.r.t the base frame {0} is

$$ {}^{0}_{n}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdots {}^{n-1}_{n}T , \quad ? \text{ How to calculate } {}^{i-1}_{i}T $$

Joint space and Cartesian Space..

An n d.o.f robot ~~Bo~~ arm may be described by a set of n joint variables that form a joint-displacement vector.

i.e. $\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$ for PUMA 560   2R-P $\Theta = \begin{bmatrix} \theta_1 \\ d_2 \\ \theta_3 \end{bmatrix}$

The dimension of the joint space = the d.o.f of the robot = the number of joints in robot

Cartesian space:

P end-effector = $\begin{bmatrix} p_x \\ p_y \\ p_z \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$ $\begin{matrix} \left.\begin{matrix} p_x \\ p_y \\ p_z \end{matrix}\right\} \text{position vector} \\ \text{Euler angles} \end{matrix}$

$\Theta \Rightarrow P$ Forward kinematics

$P \Rightarrow \Theta$ Inverse kinematics. (see previous examples)

other Topics related: Dynamics: Velocity, acceleration

$$ \tau = M(\theta)\ddot{\theta} + C(\theta\dot{\theta})\dot{\theta} + G(\theta) $$

§ 2.3 Kinematics of other Mobile Robots

① Aquatic Vehicle : thrusters, propeller

② Fly Vehicle : Helicopter Control, Fixed-Wing Control, Buoyant vehicle.

③ Space robot : Robot manipulator : advantage ? micro gravity.